# Fuzzy Logic

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Out beyond ideas of wrong-doing and right-doing,

there is a field. I'll meet you there.

When the soul lies down in that grass,

the world is too full to talk about.

Ideas, language, even the phrase each other

doesn't make any sense.

Rumi (Barks, 1995)

## Introduction

What could fuzzy logic possibly do with a quote from a 13th Century Sufi poet? Hopefully, by the end of this paper, you will see the connection. For now, trust that the whole idea of going beyond Right and Wrong, True and False, is what fuzzy logic is all about. Fuzzy logic (like Rumi, I suppose) is not a frequent topic around the break tables at psychological conferences. As a matter of fact, unless you are a specialist in the technical arena of perception modeling, you might go your entire psychological career without knowing much of anything about the subject. In a non-scientific review of many cognitive psychology undergraduate and graduate texts, I found no references to fuzzy logic in any index. In contrast, fuzzy logic is a very hot and growing topic among mathematicians, computer programmers, and engineers. Don't worry. While fuzzy logic and its applications can get extremely technical and mathematically dense, the purpose of this paper will be to introduce the reader to some historical background on the development of fuzzy logic, its basic concepts, and an overview of its application to psychology.

At its simplest, fuzzy logic is a system of logic that recognizes more than simple true and false values. Using this logic, propositions can be represented with degrees of truthfulness and falsehood. For example, the statement, "today is sunny," might be 100% true if there are no clouds, 80% true if there are a few clouds, 50% true if it is hazy, and 0% true if it is overcast all day.

The word "fuzzy" does not imply a logic that is imprecise or ill defined. On the contrary, the logic used is extremely robust mathematically and utilizes operations and techniques that are very precisely defined. Instead, "fuzzy" refers to the type of problems this methodology is good at modeling. Specifically, fuzzy logic is a powerful technique for drawing definite conclusions from complex systems that generate vague, ambiguous, or imprecise information. You might start getting an idea of why this type of logic might be a useful modeling tool in the field of psychology: Our brains are fuzzy logic experts. We make definite conclusions and decisions every moment of our lives based on vague, ambiguous, and imprecise information about our inside and outside worlds supplied by our various perception, memory, and affective subsystems.

### History and Background

Fuzzy set theory, the bedrock of fuzzy logic, was introduced by Lofti Zadeh in 1965. It was specifically designed to mathematically represent uncertainty and vagueness with formalized logical tools for dealing with the imprecision inherent in many real-world problems (Zadeh, 1965). Until this date, logic, especially in the West, was dominated by “Bivalent” set theory: statements were either true or false; things either were a member or not a member of a set. The most important feature of bivalent sets is their mutual exclusivity. As my mother always said while she held the door open for the cat standing at the threshold, “Come on, make your decision. You’re either in or you’re out. You can’t be both.” My mother did not know it, but she was expressing a basic tenet of bivalent set theory: the cat could either be in the set of objects inside the house or in the set of objects outside the house, but could not be a member of both sets.

This Yes/No, True/False, In/Out type of crispness has permeated scientific thinking since the days of Aristotle. In Western thought, systematic logic is considered to have begun with Aristotle's collection of treatises, the *Organon* (Tuccows Inc., 2002). Aristotle posited that three laws were the foundation for all valid logic: the law of identity, *A is A*; the law of contradiction, *A cannot be both A and not A*; and the law of the excluded middle, *A must be either A or not A*. The law of contradiction and the law of the excluded middle are essentially the principles of bivalent set theory that my mother expressed so much more clearly and practically: “You’re either in or you’re out” – the law of the excluded middle, and “You can’t be both” – the law of contradiction.

Even when Parminedes proposed the first version of this law (around 400 B.C.) there were strong and immediate objections: For example, Heraclitus proposed that things could be simultaneously true and not true (Brule, 1985).

In anticipation of fuzzy logic, Plato in *Phaedrus* (Liu, 1999), considers a third region beyond True and False (sounds like Rumi, doesn’t it?). Throughout the ensuing years, a few philosophers have echoed his sentiments, notably Hegel, Marx, and Engels (Brule, 1985). However, it was not until early in the 20th Century that Jan Lukasiewicz proposed a systematic alternative to the bivalent logic of Aristotle (Holmdahl & Stachowicz, 2001).

In describing a three-valued logic, Lukasiewicz asked the reader to “… believe that reality is reasonable and contradictory at the same time” (LeBlanc, 2001). The third value he proposed can best be translated as the term “possible,” and he assigned it a numeric value between True and False. He later explored four and five-valued logics, and stated that there was nothing to prevent the development of infinite-valued logic (Brule, 1985).

It was not until 1965, when Zadeh published his seminal work (Zadeh, 1965), that the notion of an infinite-valued logic took hold. He proposed an entire new set of operations and calculus of logic and showed it to be a generalization of classic logic.

### The Basics of Fuzzy Logic

Zadeh (1965) observed that most of the concepts with which humans wrestle and label experience are imprecise or "fuzzy." This is both a necessity and an advantage. For example, consider comparatively simple labels, such as *tall* and *very tall.* There is no precise boundary between these two labels; people do not carry around in their heads numeric values to distinguish the concept *very tall* from *tall.* These are what Zadeh (1973) identified as *fuzzy variables* because of their gradual progression from membership to non-membership in a fuzzy set.

Thus, the central notion of fuzzy logic is that “truth values” or “membership values” can vary continuously from, by convention, 0 to 1. In contrast, when bivalent logic is used, there are only two possible “truth values”: 0 (false) and 1 (true).

For example, consider the statement:

“Bob is old.”

Using bivalent logic, this statement would be either true or false: Bob is either old or he is not. With fuzzy logic, its truth value can be any number between 0 and 1. If Bob’s age is 75, we might assign the statement a truth value of .80. It is tempting to interpret this truth value as meaning, “There is an 80% chance that Bob is old.” A fuzzy logician would interpret the .80 truth value as meaning, “Bob’s degree of membership within the set of old people is .80.” The semantic difference is significant: the first interpretation assumes that Bob is or is not old (still caught in the law of the excluded middle); it is just that we only have an 80% chance of knowing which set he is in. By contrast, fuzzy logic supposes that Bob is “more or less” old, or some other term corresponding to the value of .80. This allows Bob to also be a member of other age groups at the same time. For instance, we might say that his degree of membership in the set of middle-aged people is .40 and his degree of membership in the set of young people is .10. In bivalent logic, this simply is not allowed.

With this foundation laid, we can briefly cover some other important characteristics of fuzzy logic as outlined by Zadeh (1992). In fuzzy logic,

- Exact reasoning is viewed as a limiting case of approximate reasoning.
- Everything is a matter of degree.
- Knowledge is interpreted as a collection of elastic, fuzzy constraints on a collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.

- Any logical system can be "fuzzified."

**Fuzzy Models**

The conceptual model of the various components of a traditional fuzzy system is shown in Figure 4 (Horstkotte & Joslyn, 1997). The first step in creating a fuzzy model of a system is to "fuzzify" the inputs. This basically means applying fuzzy membership functions to the input – assigning group memberships and membership values to input data. The second step is to use Zadeh's fuzzy set logic combined with knowledge about the system to make a set of inferences and associations between and among members in various groups. The last step is to "defuzzify" these inferences and associations and reach a decision or create some output for the system.

This basic methodology, while very simply presented and conceptually discussed, can get very complex mathematically and logically. It is used in many different ways in many different arenas. Most of its applications to date have little or nothing to do with psychology: creation of machine intelligence and expert systems, complex machine control systems, robotics, computer processor design, man-machine interfacing, weather system modeling, and endless uses in consumer products.

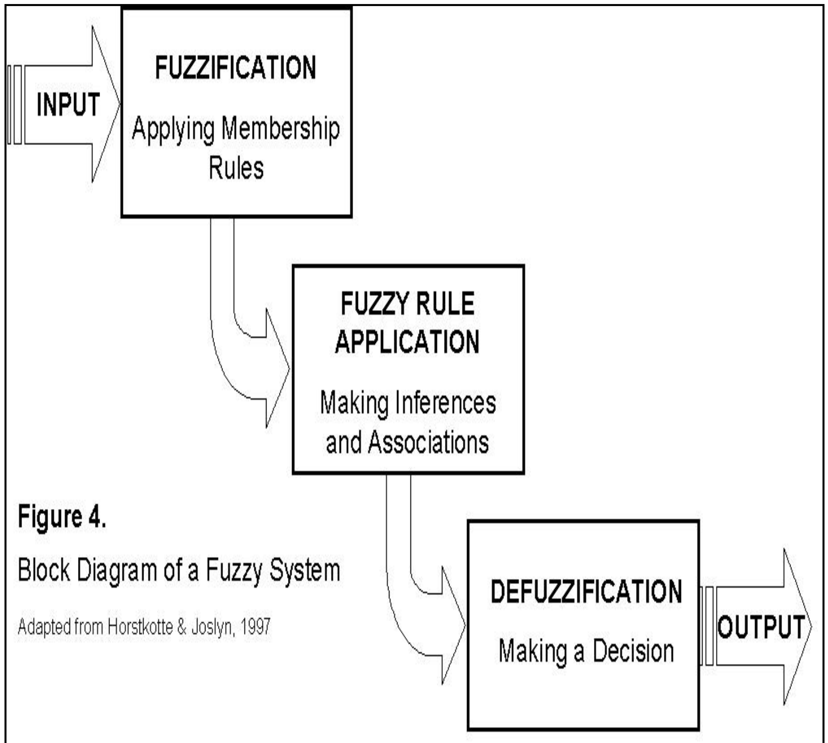


**Figure 4.**
Block Diagram of a Fuzzy System
Adapted from Horstkotte & Joslyn, 1997

**Applications in Psychology**

In psychology, fuzzy logic has been used to model complex systems, like human intelligence, perception, psychological diagnoses, or natural language processing. In other applications, fuzzy logic or approaches have been used as a way of "naturalizing" or "humanizing" a process, such as categorization and research questionnaires. Still other approaches use fuzzy logic to help decision making and making sense of "dirty" or "noisy" data. We will briefly discuss some of these applications and then spend some significant time discussing the longest running and most widely discussed application of fuzzy logic in psychology: The Fuzzy Logic Model of Perception (FLMP).

<u>Using "fuzzy" variables</u>: In 1932, Renis Likert invented a measurement method, called the Likert Scales, used in attitude surveys. They allowed answers that ranged from "strongly disagree" to "strongly agree." While technically not fuzzy sets (the choices are

still mutually exclusive), it "fuzzified" standard yes/no, agree/disagree answers and anticipated even more fuzzy approaches to measurement of preference almost 60 years later. Hesketh and colleagues (1995; 1989) have applied a true fuzzy logic graphic rating scale to the measurement of preferences for occupational sex type, prestige, and interests. In practical terms, their fuzzy variables facilitate the measurement of ranges of scores that capture individuality more accurately. As a last step, they "defuzzify" the fuzzy variables by using fuzzy-set theoretic operations (such as the union and intersection) to translate the ranges into a single score facilitating traditional psychometric analyses (Hesketh et al., 1989).

Driving behavior: Brackstone (2000) used a fuzzy logic model to more accurately model driver behavior and perception.

Expert Systems: Ohayon (1999) describes a fuzzy logic conceptual framework and the analytical possibilities of a computerized diagnostic tool to assess sleep disorders. He used over 300 interviewers who conducted over 34,000 interviews to create the database for this expert system. In another application, Shin (1998) developed a method of quantifying sleep-disordered breathing for the purpose of automating adjustments to a breathing machine. This algorithm, based on fuzzy logic, emulated the less-than-crisp kind of decision-making generally employed at the human level.

Modeling Emotion: Russell (1997) uses a fuzzy model of emotion he calls "circumplex" to build what he describes as "a fairly complete description of emotions" composed of six distinct properties.

Human Cognition: Huttenlocher and Hedges (1994) suggested a fuzzy logic approach to determining the extent to which people's mental operations correspond to formal logical rules. The classic approach to human category combination has been to identify it with formal operations on bivariant sets, as modeled with Vènn diagrams. They showed that a fuzzy logic approach more closely describes the current belief that many categories are best thought of as having a "graded" structure organized around a "prototype," with members that vary continuously in typicality from good to poor, and boundaries that are inexact and fuzzy.

## The FLMP

With his Fuzzy Logic Model of Perception (FLMP), Massaro (1987b; 1989) promoted a new paradigm for psychological research. The paradigm embraces the existence of multiple sources of information and the problem of their integration in perception. In Massaro's view the perceptual world is a rich place, full of information to be picked up, gathered, and processed at every turn.

Built on the work of Anderson (1981; 1982) , Massaro's fuzzy logic paradigm systematically explores information integration. The domains Massaro has studied and to which he has applied his FLMP are impressive in breadth and cover most of cognitive psychology; they include

- Attention (Massaro, 1985)

- Reading (Massaro, 1984, 1987a, 1998; Massaro et al., 1993)
- Letter recognition (Massaro & Friedman, 1990; Massaro & Hary, 1986)
- Speech perception (Massaro, 1987b, 1996; Massaro & Cohen, 1999; Massaro et al., 1996, 1997; Massaro & Oden, 1995)
- Visual perception (Massaro, 1987b, 1989; Massaro et al., 1996, 1997; Massaro & Friedman, 1990)
- Feature evaluation (Ellison & Massaro, 1997)

Consistently, Massaro's FLMP has provided the best fit of alternative models in each of these domains.

Within the framework of the FLMP, perceptual events are processed in accordance with a general algorithm. For each of the specific areas of interest listed above, this general FLMP structure was used, but specific membership functions, rules, and decision algorithms were created for each new application. As shown in Figure 5, the model consists of three operations: feature evaluation, feature integration, and decision. It is no mistake that the FLMP model looks very much like the conceptual model of a fuzzy system presented in figure 4. The sensory systems transduce the physical event and make available various sources of information called *features.* These continuously valued ("fuzzified") features are evaluated and matched against prototype descriptions in memory by a process that integrates individual feature values according to the specifications of the prototypes. An identification decision is then made on the basis of the relative goodness-of-match of the stimulus information with the relevant prototype descriptions. This relative goodness-of-match value thus predicts the proportion of times the stimulus is matched with the prototype or predicts a rating judgment indicating the degree to which the stimulus matches the category. A strong prediction of the FLMP is that the impact of one source of information on performance increases with increases in the ambiguity of the other available sources of information.

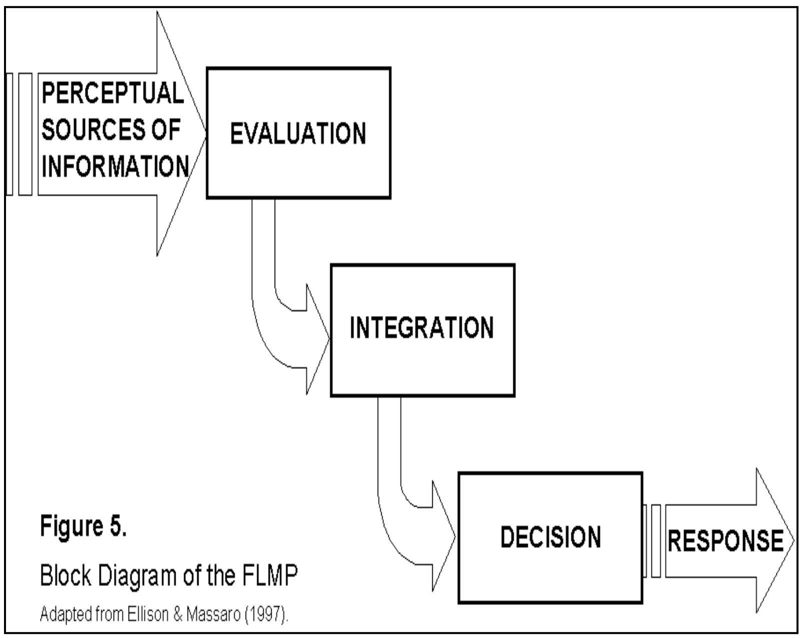

**Figure 5.**
Block Diagram of the FLMP
Adapted from Ellison & Massaro (1997).

The FLMP provides a natural account of the integration of bottom-up and top-down sources of information in processing sensory data. The major attraction of this model has been its ability to account for context dependency in perception while maintaining strict independence in the basic perceptual processes (Massaro & Oden, 1995).

## Conclusions

Fuzzy logic can be an extremely versatile and flexible tool with which to model systems that are complex, vague, and imprecise. In psychology, it has served mainly as an excellent all-purpose modeling tool for many different areas of perception, but is also beginning to be seen in other areas such as expert diagnostic systems and alternative methods of participant response processing. Its use will most likely increase as more sophisticated methods for modeling various cognitive systems are created to account for the steady advances in neurobiology. Look also for advances in artificial intelligence to be a result of the marriage of fuzzy logic and neural networked computers.

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